MAGNETOHYDRODYNAMIC(MHD) TURBULENCE AND MODELING

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Abstract

In this paper, modelling of Magnetohydrodynamics (MHD) turbulence are provided. After a short view of Magnetohydrodynamics (MHD), history of MHD described, methodology of MHD are presented.Eddy viscosity model and reynolds stress transport model are discussed. Modelling of MHD is shown with using extended with an additional scalar alpha. Alpha symbolizes tendency towards to two dimensionality. Short brief about shell models are presented. At the end of the paper comparing of some results are provided.

1 Introduction

In classical physics turbulence remains one of the significant unresolved problems. Magnetohydrodynamics Turbulence(MHD) which is primarily interest of plasma physicists and astropysicists, corcens the chaotic regimes of magnetofluid flow at high reynolds number. Magnetohydrodynamics (MHD) provides a powerful tool to analyse fluid which behavior like magnetized plasmas. First time magnetohydrodynamics word is used by Hannes Alfven in 1942 .Thomas said "At last some remarks are made about the transfer of momentum from the Sun to the planets, which is fundamental to the theory. The importance of the magnetohydrodynamic waves in this respect are pointed out." (Thomas Jefferson)(3). Indeed the MHD equations parellel in many ways to the Navier-Stokes ones and this provides us some concepts to understand of turbulence such as self-organization, cascades and closure method(7). Magnetohydrodynamics has very important role of the magnetic field in geophysical and astrophysical fluid it is generally explain like that, MHD is interaction between an electrically conducting fluid and magnetic field(3). In addition it can classify like terrestrial MHD and astrophysil MHD. Terrestrial MHD means motion of weakly conducting fluid with low magnetic Reynolds number, astrophysical MHD means motrion in a highly conducting fluid with high magnetic Reynolds number. MHD is a fluid approximation does not describe the detailed processes of plasma physics which require description of individual motions of particles like plasma frequencies, lenght scales for both ions and electrons(1). It is valid for some plasmas and liquid metals not all. Also some systems can described using MHD these are solar wind, heliosphere, Earth's magnetosphere, inertial range of plasma turbulence, neutron stars magnetospheres.

2 Methodology

Faraday's Law

$$\nabla x E + \frac{\partial B}{\partial t} = 0 \tag{1}$$

Ampere's Law

$$\nabla x B = \mu_0[j + \epsilon\frac{\partial E}{\partial t}] \tag{2}$$

$$\epsilon\frac{\partial E}{\partial t} = 0 \tag{3}$$

Coulomb's gauge

$$\nabla x B = 0 \tag{4}$$

$$\tag{5}$$

Where j is current density, μ_0 is magnetic permeability of free space on the other hand ϵ is permittivity of free space.
Ohm's Law

$$j = \sigma(E + uxB) \tag{6}$$

$$\tag{7}$$

Where σ is electrical conductivity
With combining equations we obtain induction equation.

$$\frac{\partial B}{\partial t} = \nabla x(uxB) + \mu\triangle B \tag{8}$$

$$\tag{9}$$

where μ is magnetic resistivity.
Incompressible MHD equations are

$$\frac{\partial u}{\partial t} + (u\nabla)u = -\frac{\nabla p}{\rho} + \nu\triangle u + \frac{(jxB)}{\rho} + F \tag{10}$$

$$\frac{\partial B}{\partial t} + (u\nabla)B = (B\nabla)u + \mu\triangle B \tag{11}$$

$$\nabla u = 0 = \nabla B \tag{12}$$

F is body force which can be friction, gravity or coriolis force. B can be replaced by the sclaed magnetic field which is presented below.

$$b = \frac{B}{(\rho\mu_0)^{\frac{1}{2}}} = v_a \tag{13}$$

v_a s Alfven velocity. Thus we get dimensionless parameters which are kinetic reynolds number(R_e), magnetic reynolds number(R_M), magnetic prandtl number(P_M).

$$R_e = \frac{advection}{dissipation} = \frac{UL_0}{\nu} \tag{14}$$

$$R_M = \frac{induction}{diffusion} = \frac{UL_0}{\mu} \tag{15}$$

$$P_M = \frac{\nu}{\mu} \tag{16}$$

(3) P_M is important and very small parameters. It shows us what we are working on it.
Liquid metals P_M 10^-6 .. 10^-5
Stars P_M 10^-7 .. 10^-4
Sun P_M 10^-2
When P_M is large it means poorly conducting fluids like protogalaxies, galaxies, solar wind or interstellar medium(1).

3 Reynolds Stresses Modeling

The mean momntum equation mst be closed by providing a model for the renolds stressess or by defining closed model transport equations for them. The mean transport equation can be written:

$$\frac{\partial Q}{\partial t} + U_j \frac{\partial Q}{\partial x_j} = \frac{\partial}{\partial x_j}(\frac{\Gamma_Q}{\rho}\frac{\partial Q}{\partial x_j} + \overline{u_i' q}) + S_Q \tag{17}$$

WhereρS_Q is a volume source of the scalar. Turbulent fluctuations pitch in to the transport of the mean quantity by the turbulent Reynolds flux $-\rho\overline{u_i' q}$ which must also be eveluate by some type of turbulence model. When we consider model case without external body forces, conventional turbulence models can categoriz in two classes which are eddy viscosity models(means k-ϵ model) and Reynolds stress transport model.

For the reynolds stress transport model we need to model reynolds flux term which can be modeled as shown below

$$-\overline{u_i' q'} = \frac{C_s}{\sigma_Q}\frac{K}{\epsilon} R_{ij}\frac{\partial Q}{\partial x_j}$$

C_s:Modeling Constant
σ_Q Prandtl Number
On the other hand in the eddy viscosity models, reynolds stress term is modeled with using Boussinesq hypothesis. So

$$-\overline{u_i' u_j'} = -\frac{2}{3}K\delta_{ij} + \nu_T S_{ij} \tag{18}$$

$$S_{ij} = \frac{1}{2}(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}) \tag{19}$$

In this equation ν_T means turbulent viscosity and S_{ij} is strain tensor. For the K-epsilon model turbulent viscosity can shown like

$$\nu_T = C_\mu \frac{K^2}{\epsilon}$$

So reynolds flux term is

$$-\overline{u_i' q'} = \frac{\nu_T}{\sigma_Q} \frac{\partial Q}{\partial x_i}$$

When put flux term in to the equation, we have two equation for K and Epsilon. These will use in the next chapter to obtain model of $k - \epsilon - \alpha$.

$$\frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j} = P_K - \epsilon + \frac{\partial}{\partial x_j}(\frac{\nu_T}{\sigma_Q} \frac{\partial K}{\partial x_i}) \tag{20}$$

$$\frac{\partial \epsilon}{\partial t} + U_j \frac{\partial \epsilon}{\partial x_j} = C_{\epsilon 1} \frac{\epsilon^2}{K} P_K + C_{\epsilon 2} \frac{\epsilon^2}{K} + \frac{\partial}{\partial x_j}(\frac{\nu_T}{\sigma_Q} \frac{\partial \epsilon}{\partial x_i}) \tag{21}$$

$C_{\epsilon 1}$,$C_{\epsilon 2}$,σ_K,σ_ϵ are model constants

4 Modeling of MHD Turbulence

So we obtain some equations for model k-ϵ and RST model. To create a relationship with magnetic dissipation and these equation we will define joule sink terms when we apply dimensional analyse our joule sink terms μ and μ_ϵ can presented for K and ϵ:

$$\mu = d_1 \frac{\sigma B^2}{\rho} K$$

$$\mu_\epsilon = d_2 \frac{\sigma B^2}{\rho} \epsilon$$

As we know from the previous chapter B means magnetic field. Also where $\frac{\sigma B^2}{\rho}$ is transposed magnetic time scale.

4.1 New Scalar α

New Scalar α will use in the destruction terms for models which presented in previous chapter.This is why, to provide more accurate model for joule dissipation. When scalar α implemented in Joule desctruction terms in the K, Epsilon and reynolds stress equations

,terms can be presented like:

$$\mu = \frac{2\sigma B^2}{\rho} K\alpha \tag{22}$$

$$\mu_\epsilon = C_{\epsilon\alpha}\frac{2\sigma B^2}{\rho}\epsilon\alpha \tag{23}$$

$$\mu_{ij} = \frac{2\sigma B^2}{\rho}(G(\alpha, I_{na})R_{ij} + \frac{H(\alpha)}{2}(n_i n_k R_{kj} + n_j n_k R_{ki}) \tag{24}$$

After some process on this equation we can obtain model transport equation for α

$$\frac{\partial\alpha}{\partial t} + U_k\frac{\partial\alpha}{\partial x_k} - D_\alpha = P_\alpha + \Pi_\alpha + \pi_\alpha - \mu_\alpha \tag{25}$$

In this equation where D_α is diffusion term as we obtaion before and it can be presented like For $K - \epsilon - \alpha$ model

$$\frac{\partial}{\partial x_k}(\frac{\nu_T}{\sigma_\alpha}\frac{\partial\alpha}{\partial x_k})$$

For RST-α model

$$\frac{\partial}{\partial x_k}(\frac{C_s}{\sigma_\alpha}\frac{K}{\epsilon}R_{kj}\frac{\partial\alpha}{\partial x_j})$$

ν_T:Turbulent viscosity
σ_Q:Prandtl number
P_α and Π_α: strain terms
μ_α: Descruction Term
π_α: Return to isotropy term
Nonlinear term, π_α, perform angular energy transfer.

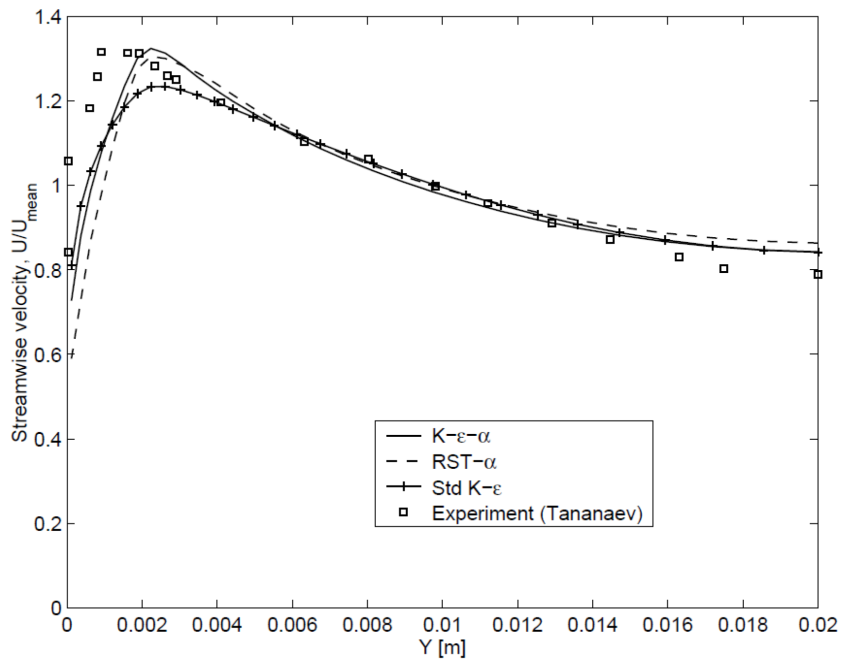

Figure 1: Comparison for models Retrieved From (1)

Figure 1 presents a channel flow velocity profile at the center. This figure shows the scalar alpha helps model more accurately. (1) Actually that can says in the all models velocity peak toke position too far from wall when compared with experiment and to be honest there is no explanation on it.

5 Shell Model

Shell models which is another touch to analytical study of fully developed turbulence, actually created by Obukhow, Desnyansky and Novikov.The main purpose of shell model for MHD turbulence is describing the statistics of isotropic and also homogeneous turbuluence in spectral space. (5) It is developed by dividing wave-vector space in discrete number of sheels whose radii grow up exponentially. (6) It is usefull model to high reynolds number. Shell models can classify like; local models, non-local models, helical models. According to Franck Plunian, local Gledzer-Yamada-Okhitani (GOY) model is most often used in 2d and 3d forms. Also it is giving perfect result in terms of spectra and energy transfer.

6 Conclusion

It is well known that a magnetic flux in an electrically conducting fluid provides electric current, which interacts with magnetic field and thus generates the Lorentz force. In this paper we discussed k-ϵ , RST models and their extended version(with α). Also i give short brief about shell models. In my opinion extended RST and k-ϵ model with alpha, certainly provides more accurate modeling of energy dissipation. That can say from derived equation α is symbolizes tendency towards to 2D. With observing $k-\epsilon-\alpha$ model we draw some idea in our mind about standard $k-\epsilon$ model and standard RST model.

7 References

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